EQUILIBRIUM

In many applications of mechanics, the sum of the forces acting on a body is zero, and a state of equilibrium exists. This apparatus is designed to hold a car body in equilibrium for a wide range of orientations during vehicle production.

3 EQUILIBRIUM

CHAPTER OUTLINE

EQUILIBRIUM

3/1 Introduction

- Statics deals primarily with the description of the force conditions necessary and sufficient to maintain the equilibrium of engineering structures.
- When a body is in equilibrium, the resultant of *all* forces acting on it is **zero**. Thus, the resultant force **R** and the resultant couple **M** are **both zero**, and we have the equilibrium equations.
- $\boldsymbol{R} = \sum \overrightarrow{\boldsymbol{F}} = \mathbf{0}$, $\sum \overrightarrow{\boldsymbol{M}_0} = \sum(\overrightarrow{\boldsymbol{F}} * \overrightarrow{\boldsymbol{d}}) = \mathbf{0}$

•Resolving each force and moment into its rectangular components leads to 6 scalar equations which also express the conditions for static equilibrium,

$$\sum F_x = 0 \quad \sum F_y = 0 \quad \sum F_z = 0$$
$$\sum M_x = 0 \quad \sum M_y = 0 \quad \sum M_z = 0$$

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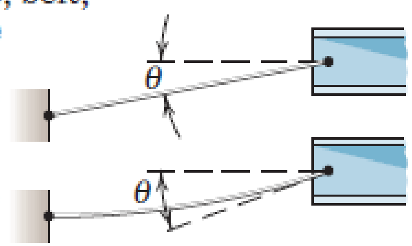
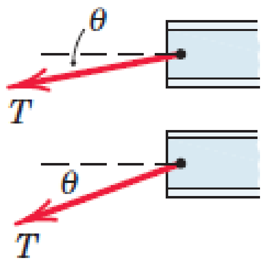
- **The free-body diagram** is the most important single step in the solution of problems in mechanics.

- **Newton's third law**, which notes the existence of an equal and opposite reaction to every action, must be carefully observed.

❖ **Reactions at Supports and Connections for a Two-Dimensional Structure**

important

Type of Contact and Force Origin	Action on Body to Be Isolated
1. Flexible cable, belt, chain, or rope Weight of cable negligible Weight of cable not negligible 	 Force exerted by a flexible cable is always a tension away from the body in the direction of the cable.

Engineering Mechanics I (Statics)

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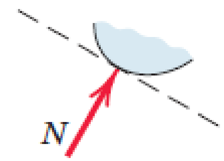
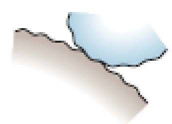
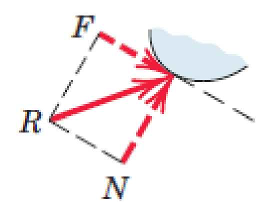
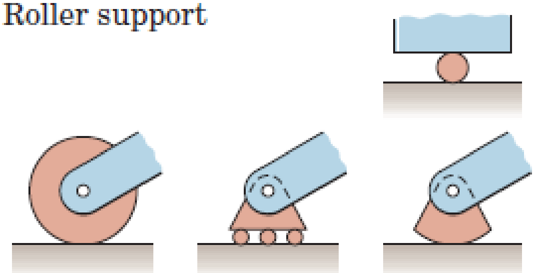
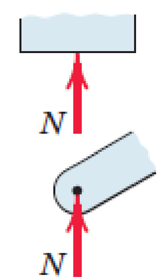
2. Smooth surfaces 	 Contact force is compressive and is normal to the surface.
3. Rough surfaces 	 Rough surfaces are capable of supporting a tangential compo-nent *F* (frictional force) as well as a normal component *N* of the resultant
4. Roller support 	 Roller, rocker, or ball support transmits a compressive force normal to the supporting surface.

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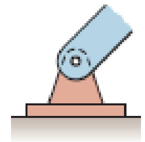
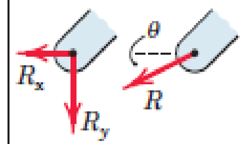
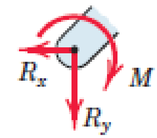
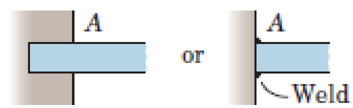
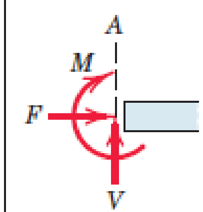
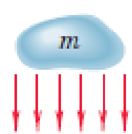
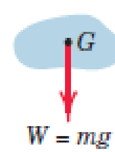
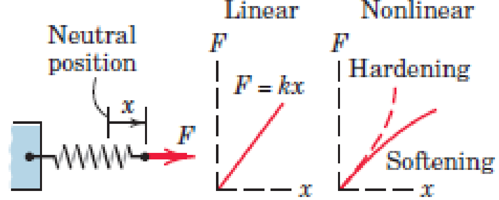
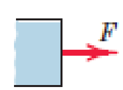
MODELING THE ACTION OF FORCES IN TWO-DIMENSIONAL ANALYSIS (*cont.*)	
Type of Contact and Force Origin	**Action on Body to Be Isolated**
6. Pin connection 	Pin free to turn  Pin not free to turn  A freely hinged pin connection is capable of supporting a force in any direction in the plane normal to the pin axis. We may either show two components R_x and R_y or a magnitude R and direction θ. A pin not free to turn also supports a couple M.
7. Built-in or fixed support 	 A built-in or fixed support is capable of supporting an axial force F, a transverse force V (shear force), and a couple M (bending moment) to prevent rotation.
8. Gravitational attraction 	 The resultant of gravitational attraction on all elements of a body of mass m is the weight $W = mg$ and acts toward the center of the earth through the center mass G.
9. Spring action 	 Spring force is tensile if spring is stretched and compressive if compressed. For a linearly elastic spring the stiffness k is the force required to deform the spring a unit distance.

❖ Construction of Free-Body Diagrams

The full procedure for drawing a free-body diagram which isolates a body or system consists of the following steps.

Step 1. Decide which system to isolate. The system chosen should usually involve one or more of the desired unknown quantities.

Step 2. Next isolate the chosen system by drawing a diagram which represents its complete external boundary.

Step 3. Identify all forces which act on the isolated system as applied by the removed contacting and attracting bodies, and represent them in their proper positions on the diagram of the isolated system.

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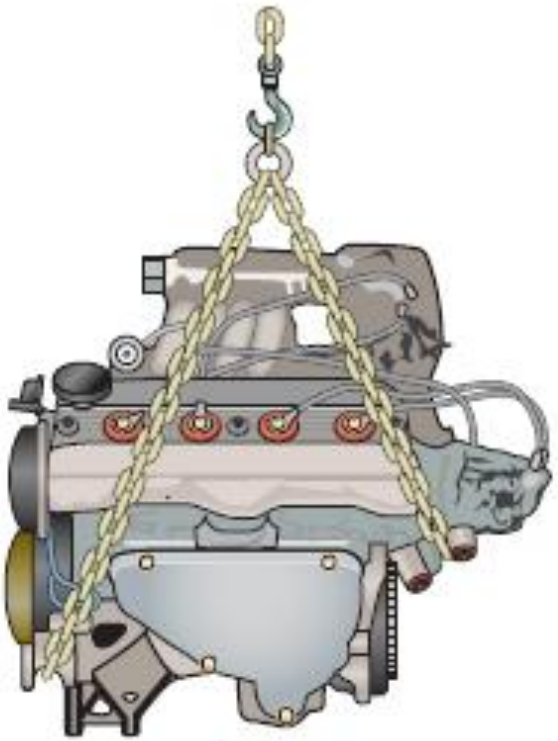
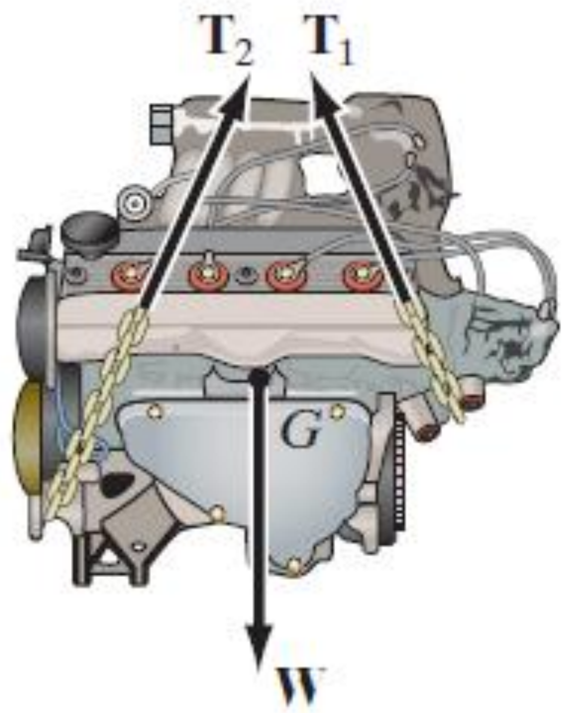
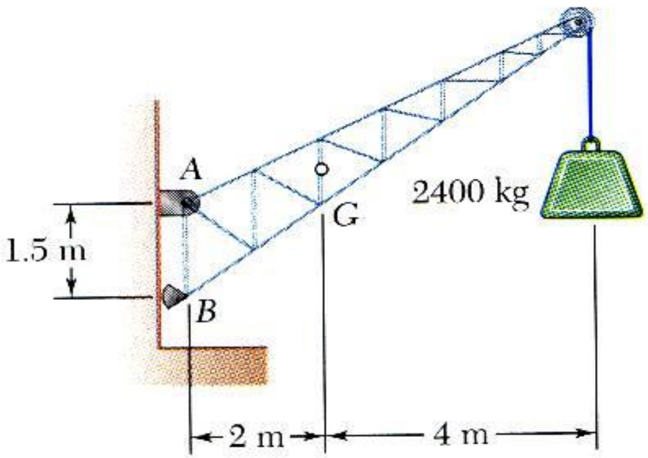
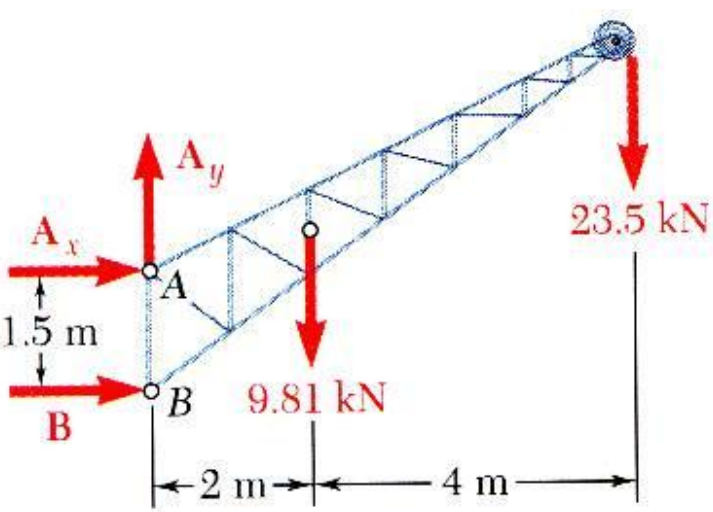


(a)



EQUILIBRIUM





First step in the static equilibrium analysis of a rigid body is identification of all forces acting on the body with a *free-body* diagram.

- Select the extent of the free-body and detach it from the ground and all other bodies.
- Indicate point of application, magnitude, and direction of external forces, including the rigid body weight.
- Indicate point of application and assumed direction of unknown applied forces. These usually consist of reactions through which the ground and other bodies oppose the possible motion of the rigid body.
- Include the dimensions necessary to compute the moments of the forces.

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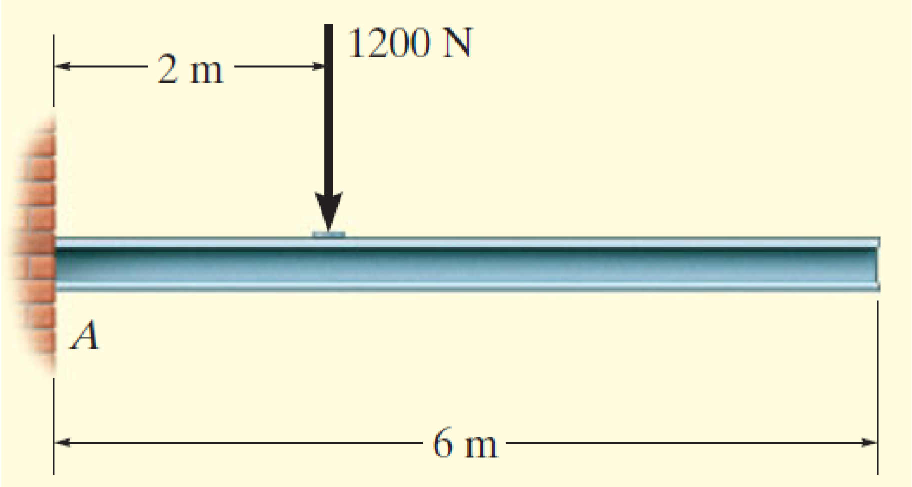
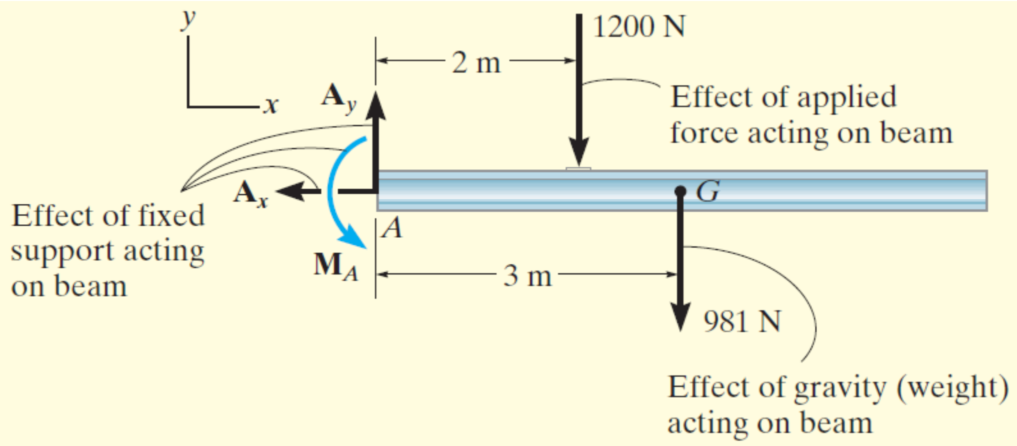
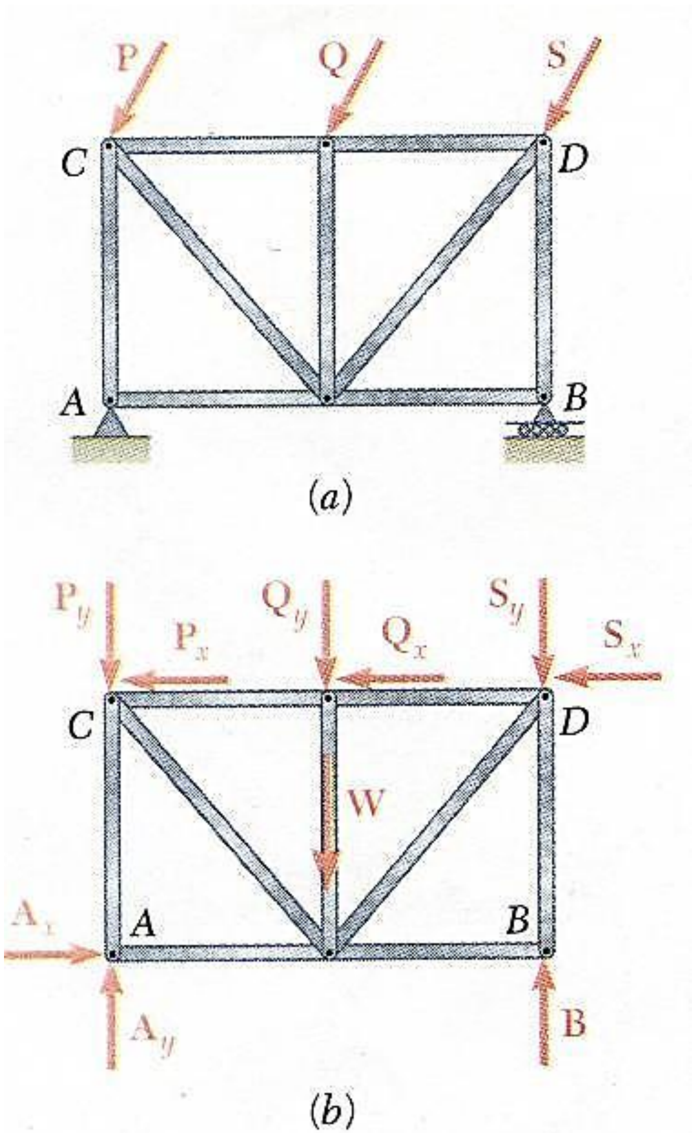






- For all forces and moments acting on a two-dimensional structure,
  $F_z = 0 \quad M_x = M_y = 0 \quad M_z = M_O$

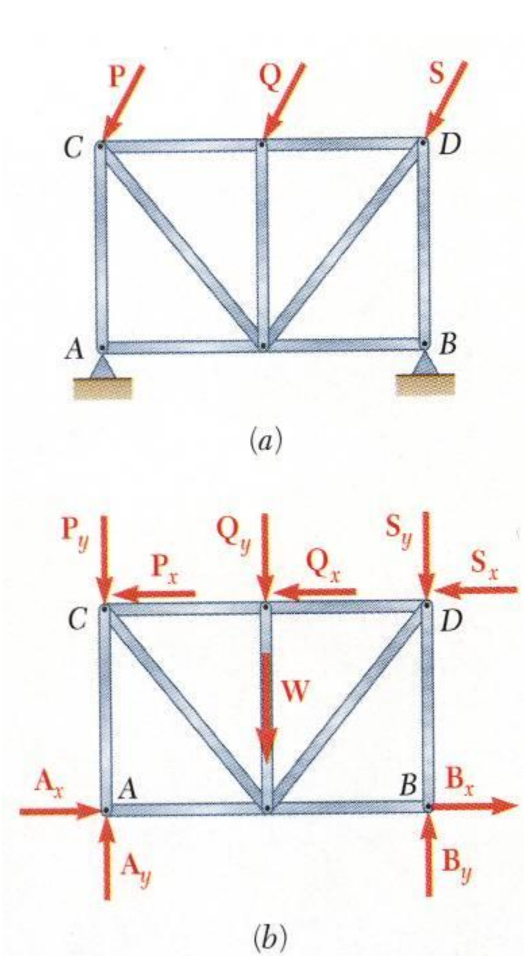
- Equations of equilibrium become
  $\sum F_x = 0 \quad \sum F_y = 0 \quad \sum M_A = 0$
  where A is any point in the plane of the structure.

- The 3 equations can be solved for no more than 3 unknowns.

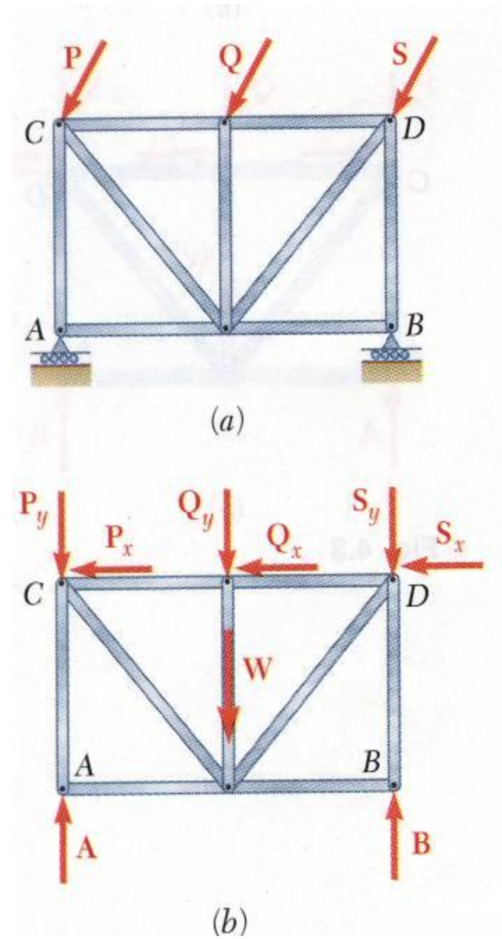
- The 3 equations can not be augmented with additional equations, but they can be replaced

$$\sum F_x = 0 \quad \sum M_A = 0 \quad \sum M_B = 0$$

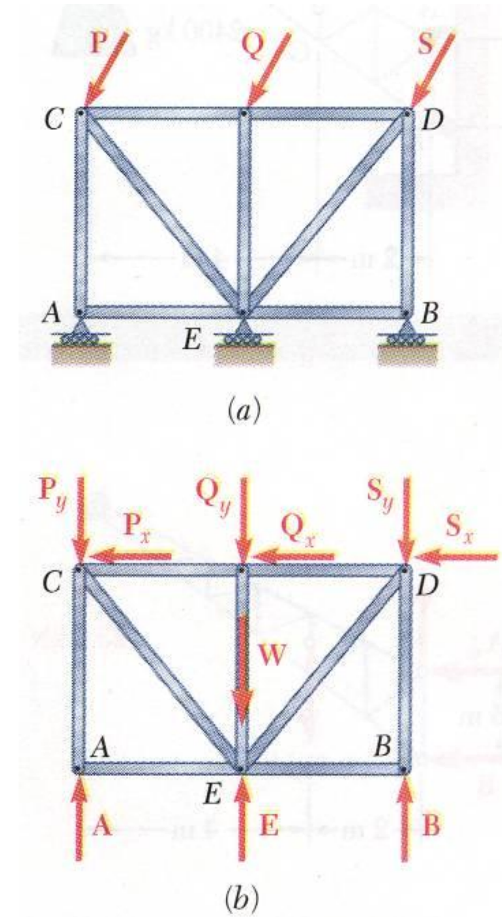
EQUILIBRIUM



- More unknowns than equations



- Fewer unknowns than equations, partially constrained

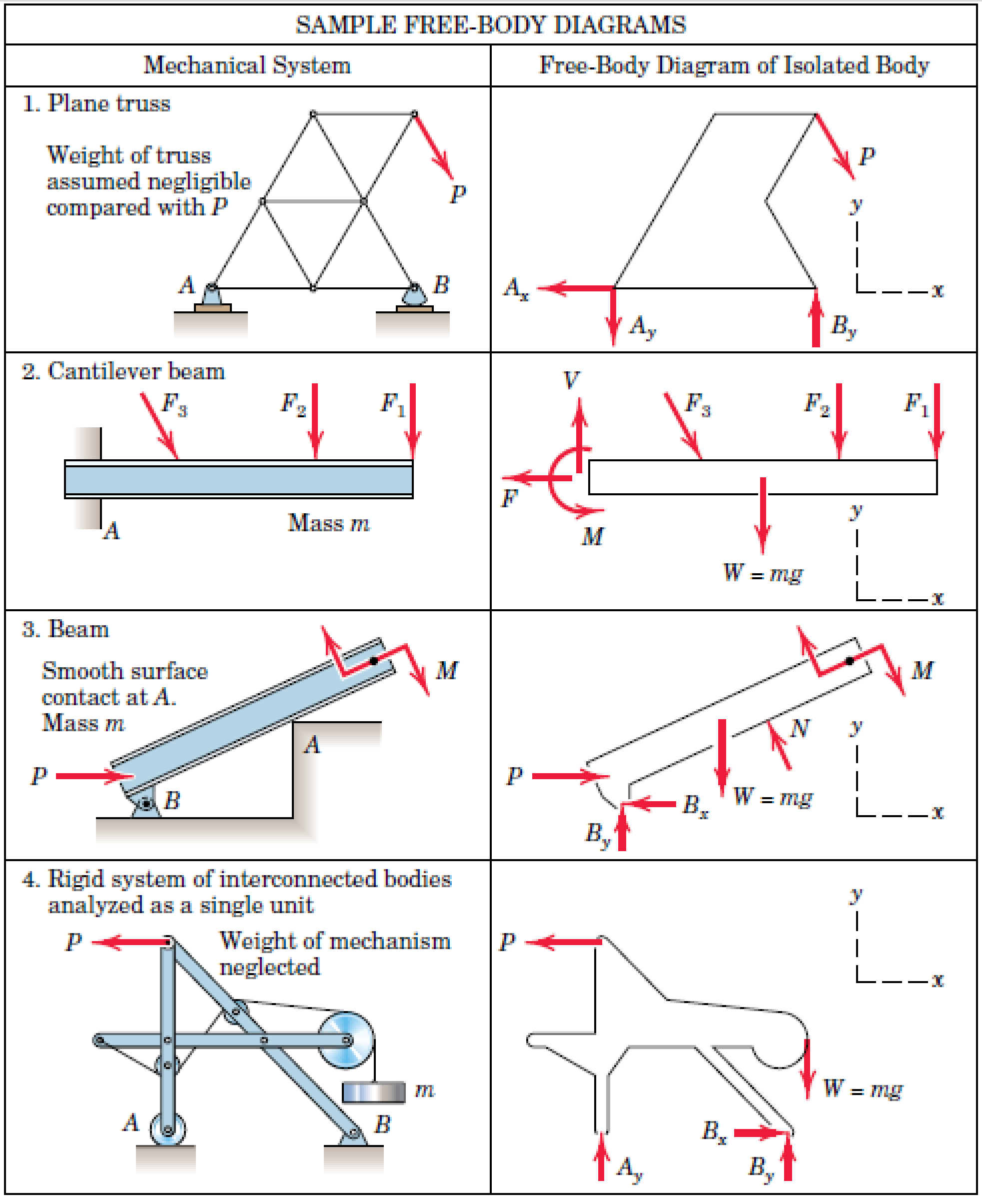


- Equal number unknowns and equations but improperly constrained

Engineering Mechanics I (Statics)

Examples of
Free-Body Diagrams

Free → Body → Diagrams



Ex:- Determine the magnitudes of the forces **C** and **T**, which, along with the other three forces shown, act on the bridge-truss joint.

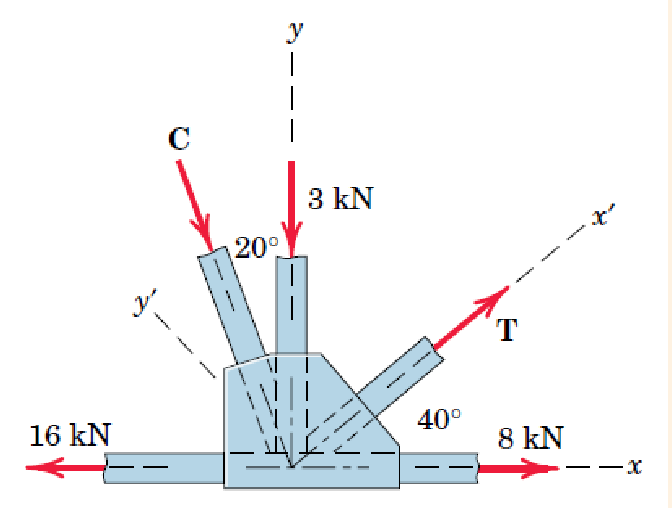
Solution.

$\sum F_x = 0$, $8 + T\cos 40 + C \sin 20 - 16 = 0$

$0.766\,T + 0.342\,C = 8$ ....(1)

$\sum F_y = 0$, $T\sin 40 - C\cos 20 - 3 = 0$

$0.643T - 0.940C = 3$ ....(2)



Simultaneous solution of Eqs. (1) and (2) produces

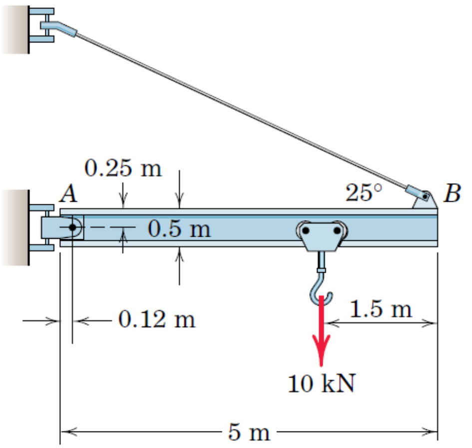
T = 9.09 kN. , C = 3.03 kN.

Ex:- Determine the magnitude **T** of the tension in the supporting cable and the magnitude of the force on the pin at A for the jib crane shown. The beam **AB** is a standard **0.5-m** I-beam with a mass of **95 kg** per meter of length.

$[\Sigma M_A = 0]$

$$(T \cos 25°)0.25 + (T \sin 25°)(5 - 0.12) - 10(5 - 1.5 - 0.12) - 4.66(2.5 - 0.12) = 0$$

from which $T = 19.61 \text{ kN}$

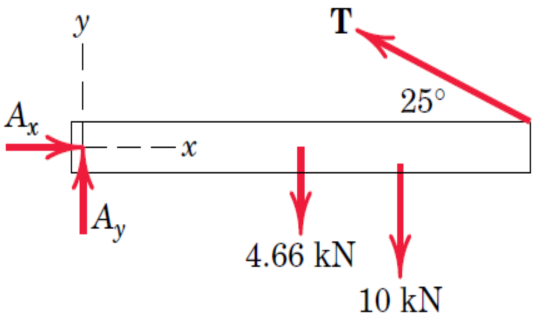


Equating the sums of forces in the x- and y-directions to zero gives

$[\Sigma F_x = 0]$ $A_x - 19.61 \cos 25° = 0$ $A_x = 17.77 \text{ kN}$

$[\Sigma F_y = 0]$ $A_y + 19.61 \sin 25° - 4.66 - 10 = 0$ $A_y = 6.37 \text{ kN}$

$[A = \sqrt{A_x^2 + A_y^2}]$ $A = \sqrt{(17.77)^2 + (6.37)^2} = 18.88 \text{ kN}$



Free-body diagram

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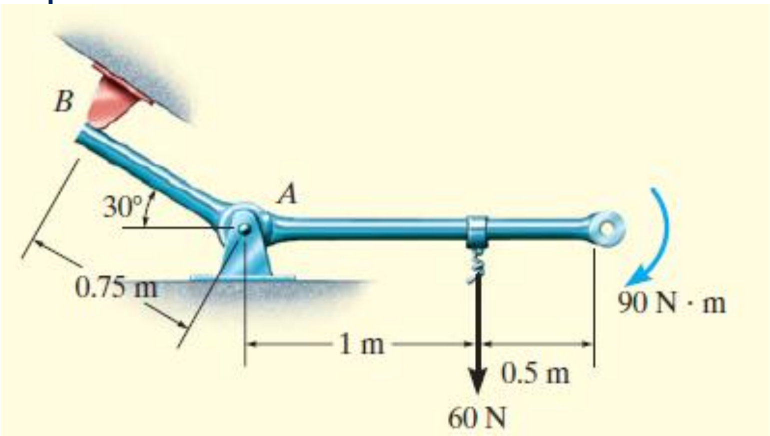
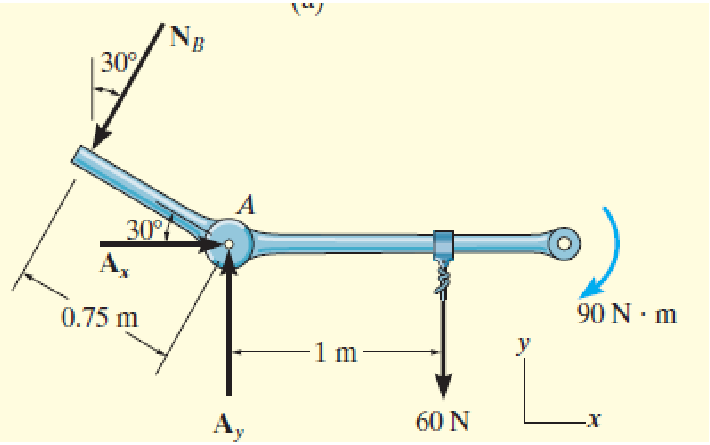
Ex:- The member shown in Fig. 5–14*a* is pin-connected at ***A*** and rests against a smooth support at ***B***. Determine the horizontal and vertical components of reaction at the pin ***A***.





Equations of Equilibrium. Summing moments about A, we obtain a direct solution for N_B,

$$\circlearrowleft + \Sigma M_A = 0; \quad -90\text{ N}\cdot\text{m} - 60\text{ N}(1\text{ m}) + N_B(0.75\text{ m}) = 0$$

$$N_B = 200\text{ N}$$

Using this result,

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad A_x - 200 \sin 30^\circ\text{ N} = 0$$

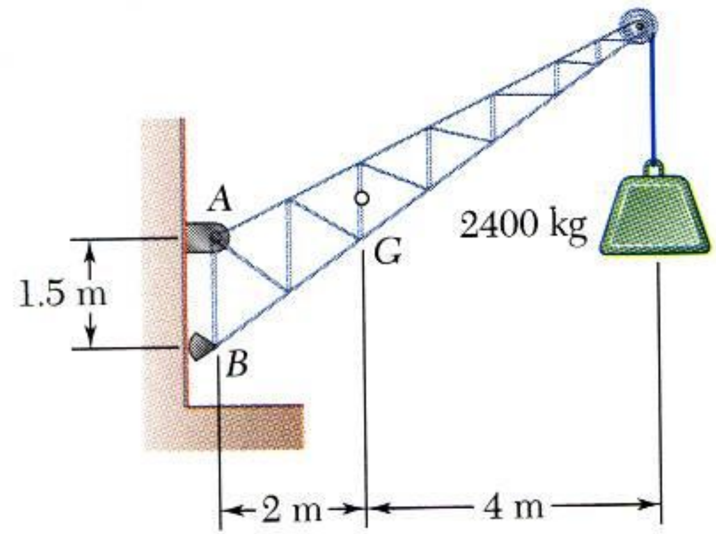
$$A_x = 100\text{ N} \qquad \textit{Ans.}$$

$$+\uparrow \Sigma F_y = 0; \quad A_y - 200 \cos 30^\circ\text{ N} - 60\text{ N} = 0$$

$$A_y = 233\text{ N} \qquad \textit{Ans.}$$

H.w.

Q1\ A fixed crane has a mass of **1000 kg** and is used to lift **a 2400** kg crate. It is held in place by a pin at ***A*** and a rocker at ***B***. The center of gravity of the crane is located at ***G***. Determine the components of the reactions at ***A* and *B***



Q2 Determine the force ***P*** required to maintain the **200-kg** engine in the position for which **30.** The diameter of the pulley at ***B*** is negligible.